# MATHEMATICS 13

## TEACHER MANUAL



CLASSROOM
ASSESSMENT
MATERIALS

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## Teacher Manual

CLASSROOM
ASSESSMENT
MATERIALS

Alberta
EDUCATION

# The Classroom Assessment Materials

## Background

The Classroom Assessment Materials Project (CAMP) was launched in 1994 in response to Alberta Education's goal of establishing and effectively communicating clear learning outcomes and high standards for each area of learning. As well, the project is a response to teachers' ongoing requests for high-quality assessment materials to use in their classrooms. CAMP also addresses the need for a common understanding of provincial standards that is frequently expressed by parents, teachers, school administrators, and other public spokespeople.

Although Alberta teachers and Alberta Education developed the Classroom Assessment Materials for teachers to use in Alberta's schools, educators from other provinces and countries have indicated that these materials have considerable potential for their jurisdictions.

## Development

Alberta Education staff have worked closely with teachers from all over Alberta to design and develop the Classroom Assessment Materials. A project advisory committee with representation for key education organizations, including the Alberta Teachers' Association, provided essential advice and direction for the overall shape and philosophy of the project. Teachers have contributed in numerous and invaluable ways. They have:

- selected resource material and data bases
- developed questions and activities
- validated materials
- offered their time and classrooms for field testing and pilot testing
- provided advice about administration and manageability
- served on revision committees
- provided advice regarding the articulation of expectations from grade to grade/course to course and across subjects
- written and revised scoring criteria
- selected examples of students' work and written commentaries about them
- confirmed that the standards represented and expressed in the final materials are appropriately demanding, faithful to *Program of Studies* expectations, and clearly expressed or illustrated

Without the dedication and professionalism of Alberta teachers, this project would not have happened.

## Purpose of the Classroom Assessment Materials

The Classroom Assessment Materials are summative assessment packages. They are designed to be used by classroom teachers to assess students' achievement of the learning outcomes specified in the *Program of Studies* relative to clearly stated standards.

The assessment activities in the CAMP materials are designed to be administrated in a classroom setting at times that suit the needs of the teacher and her or his students. The materials are not suitable for any other assessment purpose (e.g., diagnostic assessment, pre-instruction assessment, evaluation of instructional practice, system-wide assessment, program evaluation, teacher evaluation), and therefore they may not be used for any purpose imposed by any authority external to the classroom.

## Contents of Each Set of Classroom Assessment Materials

Each set of Classroom Materials contains three "documents":

- a *Teacher Manual* with complete information about the assessment activities, their relation to the *Program of Studies*, the weighting of assessment components, statements of standards, and administration instructions including scoring criteria and details for calculating students' marks
- complete *Student Materials*—all of the information, tests, and booklets that students will need for each component
- *Examples of Students' Responses* that show actual student work in relationship to the scoring criteria, along with explanatory commentary

For each grade, subject, and/or course, there are several assessment components that work together to provide teachers and parents with a broadly based portrait of a student's achievement of the expectations for students' learning at the end of that grade/course.

Each set of assessment materials includes a variety of activities—selected-response questions, short written-answer questions, extended writing activities, performance tasks such as lab experiments, problem-solving activities, and oral presentations. All activities are designed to interest students and to be of direct and practical use for teachers. All are directly related to learning outcomes from the *Program of Studies*.

## Effective Use of the Classroom Assessment Materials

Teachers may use the Classroom Assessment Materials whenever they want to find out about a student's performance in relation to set standards for the end of that grade/subject/course. The materials were developed with the following questions in mind.

- What knowledge, skills, and attitudes should a student have firmly in place before he or she moves to the next grade or course?
- How well should students completing the learning outcomes for a particular grade/subject/course do what is expected of them?
- What does acceptable work for a grade/subject/course look like?
- What does excellent work for a grade/subject/course look like?

Teachers may administer the components in whatever order suits their classroom assessment needs; however, the components are designed to be used together. Only the complete set of assessment activities will provide a portrait of how well a student has met the standards for that grade/subject/course. Teachers may photocopy the materials as their needs require.

# *Acknowledgements*

This project has come to be because of the remarkable cooperation of school jurisdictions, hundreds of teachers and principals, and thousands of students. From everyone on the project teams—thank you.

The project teams also wish to thank the following organizations without whose consultation and advice the project would not have progressed:

> Alberta Teachers' Association
> Alberta School Boards' Association
> College of Alberta School Superintendents
> Alberta Assessment Consortium
> Universities Coordinating Council
> Association canadienne-française de l'Alberta
> Public Colleges and Technical Institutes of Alberta

The Alberta Education CAMP team members from the *Curriculum Standards Branch*, *Alberta Distance Learning Centre*, *Language Services Branch*, and the *Student Evaluation Branch*.

**CAMP Project Leaders**
Frank Horvath and Elana Scraba

**CAMP Subject Coordinators**

| | |
|---|---|
| **English Language Arts** | Elana Scraba |
| **Mathematics** | Hugh Sanders |
| **Science** | Greg Hall, Greg Thomas, Bernie Galbraith |
| **Social Studies** | Doug Burns |

# *Contents*

In addition to this *Teacher Manual*, the Mathematics 13 Classroom Assessment Materials include a complete set of *Student Materials* and *Examples of Students' Responses*.
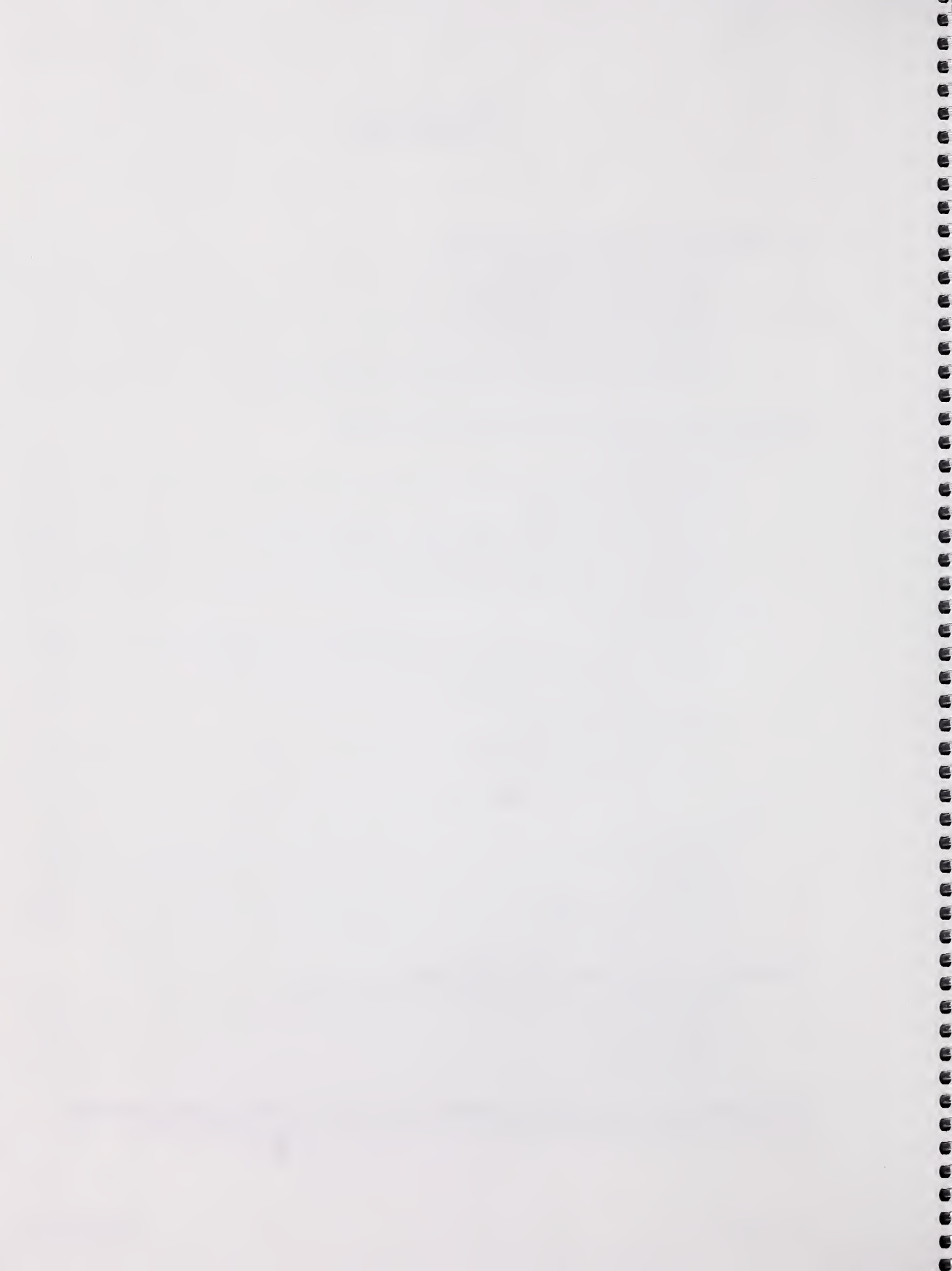
# *The Mathematics 13 Classroom Assessment*

- *Overview of the Assessment*
- *Components of the Assessment*
- *Assessment Design and Blueprint*
- *Definitions and Terminology*
- *Description of Standards*
- *Relationship of Assessment Standards to General Learner Expectations*
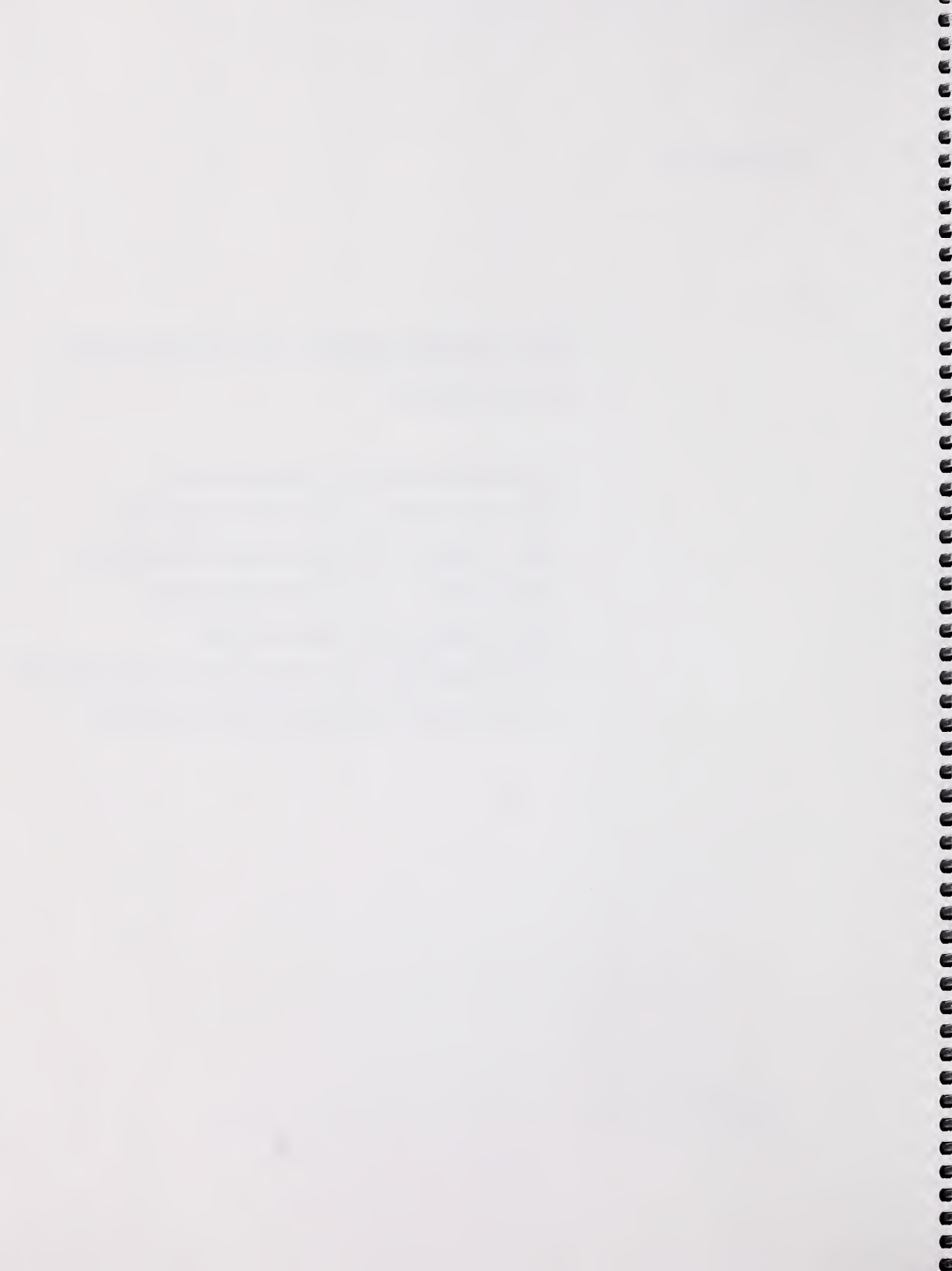
# Overview of the Assessment

The NCTM *Assessment Standards For School Mathematics (1995)* states that:

- multiple sources of assessment information should be used to evaluate student progress in mathematics
- there is a shift in the vision of learning mathematics toward investigating, formulating, representing, reasoning, and applying a variety of strategies to the solution of problems
- there is a shift in the role of teachers toward "questioning and listening" and away from "telling" students what to do
- there is a shift in the vision of evaluation toward a system based on evidence from multiple sources

In keeping with the vision of problem-solving, the assessment tasks in this package range from a more traditional end-of-course exam, including selected-response questions, numerical-response questions, and written-response questions, to tasks that are *projects* and *performance-based*.  The project and performance-assessment provide students with an opportunity to demonstrate more fully their understanding of the mathematical process.

| Type of Assessment | Part/Activity | Time | Mark Allocation | Percent Allocation |
|---|---|---|---|---|
| End-of-Course Exam | Selected Response (SR)<br>Numerical Response (NR)<br>Written Response (WR) | 1.5 h<br>(+0.5 h) | 28<br>7<br>10 | 60 |
| Performance Assessment | 1. Toothpick Triangle Patterns (Coordinate Geometry and Graphing)<br><br>2. Volleyball Follies (Statistics)<br><br>3. The Big Swallow (Number Skills) | 1.5 h<br>(+0.5 h) | 6<br><br><br>6<br><br>6 | 15 |
| Project | Statistical Inquiry | 5 h<br>(+0.5 h) | 30 | 25 |
| | **Total** | 8 h | 93 | 100 |

## Components of the Assessment

The Mathematics 13 assessment consists of the following components:

- An end-of-course test consisting of 28 selected-response, 7 numerical-response, and 2 written-response questions.
- A performance assessment consisting of 3 tasks, which relate to Relations, Coordinate Geometry and Graphing, Statistics, and Number Skills. The test is to be completed by students individually.
- A project that relates to Statistics and is designed to be completed by students over a longer period of time. Students may work independently or in groups, but each student is responsible for their own presentation.

# Assessment Design and Blueprint

| General Outcomes (as described in the assessment standards section) | Conceptual Understanding | Procedural Knowledge | Problem-Solving Contexts |
|---|---|---|---|
| Determine equivalent forms of rational numbers and of expressions with integral exponents. | WR 1 PA 3 | SR 1, 2, 3, 23 WR 1 PA 3 | SR 2, 3 NR 6 WR 1 PA 3 |
| Perform operations on polynomials, factor polynomials, and use the terminology associated with polynomials to express ideas and solve mathematical problems related to polynomial operations and factoring. | SR 13 NR 4 WR 2 | SR 9, 10, 11, 12, 14 NR 3 WR 2 | NR 3 WR 2 |
| Connect variables, tables of values, and equations to graphical representations and describe and analyze relationships between two variables, with emphasis on linear functions that model real-world contexts. | SR 4, 7, 22 NR 7 PA 1 | SR 4, 5, 6, 7, 8, 20, 21 NR 2 PA 1 | PA 1 |
| Describe the properties of triangles formed when parallel lines are intersected by a transversal. Specify and describe the conditions under which triangles may be similar or congruent, and use these conditions to solve problems. | SR 15, 16, 17, 18, 19 | SR 16, 17, 18, 19 NR 5 | |
| Organize, present, and analyze data collected from a suitable sample of a population to draw inferences about the population. | SR 24, 25, 26, 27, 28 PA 2 Project | SR 26 NR 1 PA 2 Project | NR 1 PA 2 Project |

# Definitions and Terminology

The Alberta Education Course of Studies for Mathematics 13 is defined by the learner expectations, which state what the student is expected to know and be able to do.

A standard is a reference point used in planning and evaluation. A standard defines the quality of a performance, product, or service that is considered acceptable or that is considered to be excellent.

## Definitions

In evaluating educational performance, the following standards apply:

• *curriculum* and *assessment standards* apply to the assessment of individual students

• *achievement standards* apply to the assessment of student populations

In this document, only curriculum and assessment standards will be discussed.

## Curriculum Standards

A *curriculum standard* is a set of learner expectations for a module, course, or grade level of a program. The curriculum standards for Mathematics 13 are defined by the general learner expectations outlined on pages 3 and 4 of the Course of Studies and the specific learner expectations outlined on pages 5 to 12 of the Course of Studies.

## Learner Expectations

*General learner expectations and outcomes* are concise statements identifying what it is that students are expected to know and be able to do upon completion of a module, course, or grade level of a program.

*Specific learner expectations* are the component knowledge, skills, and attitudes that contribute to general learner expectations. Specific learner expectations identify a range of contexts in which the general learner expectations apply.

**Assessment Standards**  *Assessment standards* are the criteria used for judging individual student achievement relative to the curriculum standards, and are noted in the Description of Standards section of this document. Assessment standards have three components:

- *Criteria* are descriptors of student performance that indicate a standard has been met. Specific criteria may take the form of a set of descriptors used in assessing performance on a particular task.

- *Examples* are evidence of the quality of student work.

- *Guidelines of assessment and reporting* consist of information and advice for educators, to ensure consistency in assessment and reporting.

**Assessment Instrument**  An *Assessment Instrument* is a group of questions or tasks given to students to ascertain whether they have met the requirements of the acceptable standard or the standard of excellence.

**Blueprint**  A *blueprint* is a classification of the questions or tasks making up a particular assessment instrument. The classification may be in terms of standards (acceptable or excellent), content sections (either topics or titles), or mathematical understandings (concepts, procedures, problem-solving skills), or any other such classification.

**Scoring Criteria**  The *scoring criteria* is a set of descriptions of qualities of work for awarding marks on a particular extended-response question.

**Conceptual Understanding**  *Conceptual understanding expectations* are characterized by the use of such action verbs as defining, demonstrating, describing, developing, establishing, explaining, giving, identifying, illustrating, linking, recognizing, and representing.

**Procedural Knowledge**  *Procedural knowledge expectations* are characterized by the use of such action verbs as approximating, calculating, constructing, estimating, factoring, locating, rationalizing, simplifying, sketching, solving, using, and verifying.

## Problem-Solving Contexts

The *Problem-Solving Contexts* refer to the construction of models describing applications in a broad range of contexts. They also refer to the combination of algorithms to determine more complicated solutions and the use of appropriate technology to approximate difficult models or to carry out long calculation procedures. They also refer to the investigation of phenomena, the collection and interpretation of data in various contexts, and the transfer of knowledge from mathematics to other areas of human endeavor. Problem-solving expectations are characterized by the use of such action verbs as adapting, analyzing, combining, communicating, comparing, connecting, constructing, deriving, evaluating, fitting, investigating, justifying, modelling, proving, reconstructing, and translating.

## *Description of Standards*

The content specified in the Course of Studies for Mathematics 13 consists of both required and elective material. The required content comprises 80 percent of the course and contains the concepts, skills, and attitudes that all students are expected to acquire. As well, the required portion includes specific expectations for problem solving and the use of technology. The additional elective materials, which comprise 20 percent of the course, provide for enrichment, remediation, innovative or experimental presentations and/or activities. For complete details of the Mathematics 13 program structure, refer to the Course of Studies.

The Course of Studies also states that "evaluation of students in the Senior High School Mathematics program will involve assessment of the level of achievement of **all of the learner expectations, including concepts, skills, and attitudes, as well as problem-solving and technological expectations.**"

The assessment standards for Mathematics 13 include an acceptable and an excellent level of performance. Students' performance should be measured on a range of tasks, some designed to focus on routine and obvious tasks in familiar contexts and others designed to focus on non-routine tasks in unfamiliar contexts.

**Acceptable Standard**

The *acceptable standard* of achievement in Mathematics 13 is met by students who receive a course mark between and including 50 percent and 79 percent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific learner expectations defined in the Mathematics 13 Course of Studies. These students can apply this knowledge to a limited range of familiar problem contexts. The *acceptable standard* identifies the degree to which learning expectations must be met for students to be successful at the next level of learning.

**Standard of Excellence**

The *standard of excellence* of achievement for Mathematics 13 is met by students who receive a course mark at or above 80 percent. Typically, these students have gained a breadth and depth of knowledge of the concepts and procedures, and the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts. This standard signifies high-quality performance relative to the general and specific learner expectations in the Mathematics 13 Course of Studies. The *standard of excellence* identifies the degree to which learning expectations must be met for students to excel at a next level of learning.

**Students have reached the acceptable standard** if they are able to consistently complete acceptable work on routine and obvious tasks in familiar contexts.

**Students have reached the standard of excellence** if they are able to consistently complete excellent work on routine and obvious tasks in familiar contexts and acceptable work on non-routine tasks in unfamiliar contexts.

Details of the relationship of assessment standards to curricular learner expectations follow.

# *Relationship of Assessment Standards to General Outcomes*

| Number Skills |
|---|
| ***General Outcome*** <br> *Students can determine equivalent forms of rational numbers and of expressions with integral exponents.* |

| **Acceptable Standard** | **Standard of Excellence** |
|---|---|
| *The student can:* | *The student can also:* |
| • perform calculations, recognize, and explain the equivalent forms of rational numbers (fraction, decimal, and percent) <br> • convert from one form to another <br> • explain and demonstrate how equivalent forms can be used to order rational numbers <br><br>     Show and explain how you could use equivalent forms to write the following numbers in ascending order: <br><br>     $-0.5, \ 18\frac{1}{4}\%, \ 1\frac{1}{2}, \ 2.5, \ \frac{15}{2}, \ -5, \ 4$ | |
| • perform the operations of addition, subtraction, multiplication, and division on rational numbers | • explain the operations of addition, subtraction, multiplication, and division on rational numbers |
| • perform operations on expressions involving exponents | • explain operations on expressions involving exponents |

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*Continued*

| Number Skills | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |
| • perform multiple operations involving the rules for order of operations<br><br>    Simplify: $3(12-4)^2 - 5 \times 6$<br><br>    Simplify: $\left(3\frac{1}{2}\right)^2 - \frac{1}{5} \div \frac{4}{3}$<br><br>• solve problems involving rational numbers and explain the solution in written form<br>• solve problems that involve the use of percents in a contextual setting (i.e., sales tax, GST, discounts, etc.) and explain the solution in written form<br>• estimate numerical squares and square roots<br>• calculate numerical squares and square roots, using technology<br>• evaluate powers with integral bases and integral exponents<br><br>    Simplify: $2^7, 3^0, 2^{-3}$<br><br>• write an equivalent expression for powers with integral bases and integral exponents involving the use of the laws of exponents<br><br>    Simplify: $(2^3)(2^5)$<br><br>• perform operations involving powers<br><br>    Simplify: $2^2 + 2^5$<br>    Simplify: $(2^3)(3^2)$ | • estimate numerical squares and square roots, and justify the result<br><br><br><br><br><br>• perform multiple operations involving powers, without using technology<br><br>    Simplify: $(2^3)(3^2) \div 3 + 2^{-1}$ |

## Polynomials and Factoring

### General Outcome

*Students can perform operations on polynomials, factor polynomials, and use the terminology associated with polynomials to express ideas and solve mathematical problems related to polynomial operations and factoring.*

| Acceptable Standard | Standard of Excellence |
|---|---|
| *The student can:* | *The student can also:* |

**Acceptable Standard**

*The student can:*

- write polynomial expressions to describe situations

    Sam earns $5 an hour regular pay and $8.50 an hour for overtime pay. Write a mathematical expression that describes Sam's pay. Define the variables used.

- use the language associated with powers and polynomials (exponent, base, variable, coefficient, constant, term, monomial, binomial, trinomial, like terms, degree)

- classify polynomials that are written in simplified form according to number of variables, number of terms, and degree

    Classify the following polynomials by number of variables, number of terms, and degree:

    $\frac{1}{2}x + 10$

    $5a^2b + \frac{7}{4}ab - 3ab^2$
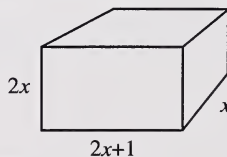
**Standard of Excellence**

*The student can also:*

- classify polynomials that are not written in simplified form according to number of variables, number of terms, and degree

    Classify the following polynomials by number of variables, number of terms, and degree:

    The area of a rectangle: $w(2w + 1)$

    The volume of the following rectangular prism: $x(2x)(2x + 1)$



*Continued*

| **Polynomials and Factoring** | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |
| • evaluate a polynomial, given integral or rational replacements for the variables<br><br>Determine the value of $y = -\dfrac{3}{4}x + 5$<br><br>when $x = -3$ and when $x = \dfrac{1}{2}$.<br><br>• verify the multiplication, division, and power laws of exponents<br><br>Verify the multiplication law<br>$a^m \times a^n = a^{m+n}$ when $m = 2$ and $n = 3$.<br><br>i.e., $a^2 \times a^3 = (a \times a) \times (a \times a \times a)$<br>$= a \times a \times a \times a \times a = a^5$ | • evaluate a polynomial that applies to real-world phenomena, given integral or rational replacements for the variables.<br><br>Derive a formula and determine the volume of a soup can that has a diameter of 5 cm and a height of 11.2 cm. |
| • apply the multiplication, division, and power laws of exponents to expressions involving powers of monomials in one variable<br><br>Perform the operation(s) to simplify each expression. State your solution using positive exponents only!<br>    a. $(2x^3)^2$<br>    b. $(2x^7)(-4x^{-5})$<br>    c. $\dfrac{-12x^2}{14x}$<br>    d. $(x^3)(2x)^4$<br>    e. How many 5 cm cubes can be placed inside a shoe box with dimensions $10 \text{ cm} \times 15 \text{ cm} \times 30 \text{ cm}$? | • apply the multiplication, division, and power laws of exponents to expressions involving powers of monomials in two or more variables<br><br>Simplify the following expression. State your solution using positive exponents only.<br><br>$(2xy^2z)(-3x^2y^{-3})^2$ |

*Continued*

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### Polynomials and Factoring

| Acceptable Standard | Standard of Excellence |
|---|---|
| • add and subtract polynomials involving one or two variables<br><br>Simplify: $(x^2 + 2x - 1) - (2x^2 - 4)$<br><br>Write an expression in simplified form for the perimeter of the following figure:<br><br><br><br>• multiply a polynomial by a monomial<br><br>Simplify: $-2x^2(5x^2 - 6x + 7)$<br><br>• determine the product of two binomials<br><br>Simplify the following:<br>$(x + 4)(x + 8)$<br>$(3x + 5y)(2x - y)$<br><br>• determine and recognize the product of the square of a binomial and the product of conjugate binomials<br><br>Simplify the following:<br>$(3t - 5)^2$<br><br>Determine the product of $2x - 5y$ and its conjugate. | • determine the product of two binomials that involve two variables with exponents other than one<br><br>Simplify the following:<br>$(3y^2 - 2x)(4y^2 - 5x)$ |
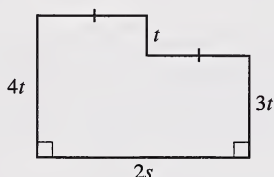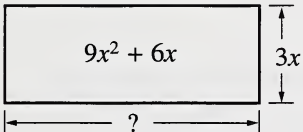
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## Polynomials and Factoring

| **Acceptable Standard** | **Standard of Excellence** |
|---|---|
| • divide polynomials by monomials | |

Simplify: $\dfrac{12x^4 - 16x^3 + 4x}{4x}$

Determine the length of a rectangle if its area is $9x^2 + 6x$ and its width is $3x$.

$9x^2 + 6x$  $3x$  ?

| | |
|---|---|
| • perform multiple operations on polynomials, using any combination of addition, subtraction, and multiplication | • perform more complex multiple operations on polynomials, using any combination of addition, subtraction, and multiplication |

Simplify:  $2x(x - 4) - 3(4x - 7)$       Simplify: $5(x^2 - 3)^2 - 3(x^2 + 2)$
$\phantom{Simplify:} 3(2a - 5)^2$

• explain that to factor a polynomial means to express it as a product of two or more expressions

Write 24 as the product of two factors.
Write 24 as the product of three factors.

Which of the following are expressed in factored form?
   $2x(x + 5) (3x - 2) - (x + 5)$
   $2x^2 + 7x - 3$
   $(x + 5) (x - 3)$

Explain why $3x$ is a factor of
$27x^4 - 12x$.

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*Continued*

| Polynomials and Factoring | |
|---|---|
| • factor polynomials that have a monomial common factor<br><br>Factor $4x^2 + 8x$ by extracting the greatest common factor.<br><br>The total surface area of a cylinder is given by $2\pi r^2 + 2\pi rh$. Express this in a completely factored form.<br><br>Factor completely: $24x^2 y^4 - 18xy^2$. | • factor polynomials with non-integral coefficients that have a common monomial factor<br><br>Factor: $\dfrac{1}{2}x^2 y^2 + 4x + y^5$ |
| • factor trinomials of the form $ax^2 + bx + c$, where $a = 1$, and $a$ $b$ and $c$ are integers, or where $a = 1$ after a common factor is removed<br><br>Factor completely:<br>$x^2 + 7x + 12$<br>$t^2 - t - 72$<br>$2x^2 + 14x + 24$ | • factor trinomials of the form $ax^2 + bx + c$, where $a = 1$, that involve two variables<br><br>Factor completely: $x^2 - 4xy - 45y^2$ |
| • determine and recognize the factors of perfect square trinomials<br><br>The product of two identical binomials is $x^2 - 12x + 36$. What are the two identical binomial factors? | • determine and recognize the factors of perfect square trinomials in two variables<br><br>Factor completely: $x^2 - 4xy + 4y^2$. |
| | • determine values that satisfy conditions for a perfect square trinomial<br><br>For what value(s) of $k$ will $m^2 + km + 64$ be a perfect square trinomial? |

*Continued*

*Continued*

| *Polynomials and Factoring* | |
| --- | --- |
| **Acceptable Standard** | **Standard of Excellence** |
| • determine and recognize the factors of difference of squares | • determine and recognize the factors of difference of squares in two variables |
|     Factor completely:<br>$t^2 - 16$<br>$9y^2 - 25$ |     Factor completely: $4x^4y^2 - 49$. |
| | • factor polynomials that require a combination of methods in order to be fully factored |
| |     Factor completely:<br>$5x^2 - 80$<br>$2t^2 + 6t - 80$<br>$3x^3 - 27x^2 + 60x$ |
| | • factor polynomials in two variables that require a combination of methods in order to be fully factored |
| |     Factor completely:<br>$4a^2 + 4ab - 80b^2$ |
| • use factoring to solve problems involving everyday occurrences | • use factoring to analyze problems involving everyday occurrences |

| Coordinate Geometry and Graphing | |
|---|---|
| **General Outcome** | |
| Students can connect variables and equations to graphical representations and describe and analyze relationships between two variables, with emphasis on linear functions that model real-world contexts. | |

| Acceptable Standard | Standard of Excellence |
|---|---|
| *The student can:* | *The student can also:* |

**Acceptable Standard**

*The student can:*

- describe the concept of slope
- determine the slope of a horizontal line, a vertical line, or an oblique line, given its graph
- explain why the slope of a horizontal line is zero and why the slope of a vertical line is undefined
- identify and draw examples of lines that have positive, negative, zero, and undefined slopes
- draw the line with a given slope passing through a given point
- determine the slope of a line passing through two given points, using the slope formula
- apply the slope formula to real-life problems

 A road rises 20 m for every horizontal change of 200 m. What is the slope of the road?

 Determine the slope of the roof shown below.
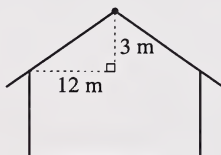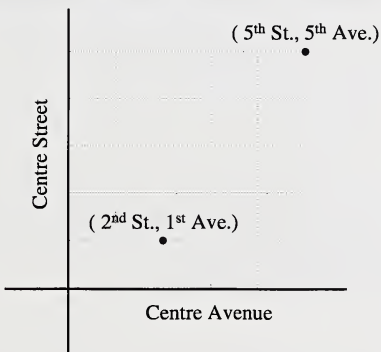


- demonstrate that parallel lines have the same slope

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| **Coordinate Geometry and Graphing** | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |

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- demonstrate that the slopes of perpendicular lines are negative reciprocals of each other
- determine the slope of a line parallel or perpendicular to another line whose slope is given
- determine the length of a horizontal or vertical line segment, given its graph
- determine the length of an oblique line segment, given its graph, using the Pythagorean relation
- determine the distance between any two given points
- solve problems that involve determining the distance between two points

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- solve problems that involve the relationships between the slopes of parallel and perpendicular lines

Given $A(5, 6)$, $B(-2, 3)$, and $C(7, -4)$, show that $\triangle ABC$ is a right-angled triangle.  Determine the coordinates of the vertex of the right angle in this triangle.

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( 5th St., 5th Ave.)

Centre Street

( 2nd St., 1st Ave.)

Centre Avenue

Above is a map of a section of a city.  If a cab had to drive from the corner of 2nd Street and 1st Avenue to the corner of 5th Street and 5th Avenue, what would be the shortest distance by road between the two corners?

If a bird travelled between the same two corners, what would be its shortest distance?

*Continued*

| Coordinate Geometry and Graphing | |
| --- | --- |
| **Acceptable Standard** | **Standard of Excellence** |
| • locate and label the coordinates of the midpoint of a line segment, given its graph | |
| • determine the midpoint of a line segment, given its endpoints | • determine the coordinates of the endpoint of a line segment, given the midpoint of the segment and the other endpoint |
| | • solve problems that involve a combination of slope, distance, and/or midpoint<br><br>Show by<br>(1) distance formula<br>(2) midpoint formula<br><br>that the point (6, 3) is the midpoint of line segment $\overline{AB}$, given $A(2, 1)$ and $B(10, 5)$. |
| • find sets of ordered pairs that satisfy a linear equation and use them to sketch the graph of the relation | |
| • determine whether a line passes through a given point, given the equation of the line | • verify algebraically and graphically whether a line passes through a given point, given the equation of the line |
| Determine whether the point (4, –5) lies on the line given by $y = \dfrac{3}{4}x - 8$. | Verify algebraically and graphically that (4, –5) lies on the line given by $y = \dfrac{3}{4}x - 8$. |

*Continued*

*Continued*

| Coordinate Geometry and Graphing | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |
| • identify the $x$- intercept and $y$-intercept of a line, given its graph<br>• draw the graph of a line, given its $x$- and $y$-intercept<br>• express linear equations in the slope intercept form $y = mx + b$<br>• describe the effects of the changing of the parameters $m$ and $b$ in $y = mx + b$<br>• graph linear equations, using the slope-intercept form<br>• write the equation of a line in the form $y = mx + b$, given the slope and the $y$-intercept, or given its graph<br>• recognize and graph equations of lines that are parallel to the $x$- axis or the $y$-axis. | |

| Relations |
|---|
| **General Outcome** |
| *Students can describe everyday occurrences using graphs, tables of values, and equations, and make connections between these different representations.* |

| **Acceptable Standard** | **Standard of Excellence** |
|---|---|
| *The student can:* | *The student can also:* |
| • identify the independent and dependent variables in a relation | |
| • sketch graphs that describe everyday occurrences | • sketch graphs that model two different situations that share a common independent variable |
| The heart rate of a student varies over a school day. Write a story that describes and explains the changes in heart rate over time and then draw a graph that represents this story. Label the axes. | The number of cars in the teachers' and students' parking lots varies over the school day. Draw a grid with two vertical axes, one representing the number of teachers' cars and the other representing the number of students' cars. Use the horizontal axis to represent various times during the day. Sketch two graphs representing how the number of cars vary and then explain your graphs in words. |
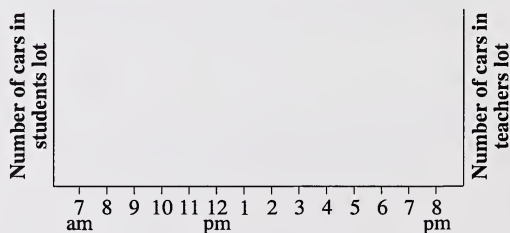
*Continued*

*Continued*

| Relations | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |

**Acceptable Standard**

- read and interpret graphs drawn to describe real-life relationships

  The following graph displays the relationship between the time of day and noise level in a school cafeteria.

  Write a story that explains the features of this graph.

*Continued*

| Relations | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |

### Acceptable Standard

- interpolate and extrapolate to find information from given graphs and tables of values

  The following table of values shows the distance a car has travelled in various amounts of time.

  | Time (minutes) | Distance (kilometres) |
  |:---:|:---:|
  | 0 | 0 |
  | 6 | 5 |
  | 15 | 12.5 |
  | 20 | 16.7 |
  | 45 | 37.5 |

  I. Use the table to sketch the graph of this relation.

  II. Use your graph to determine:

     a. the distance travelled in 30 minutes
     b. the time it takes to travel 30 km
     c. the distance travelled in one hour
     d. the speed of the vehicle

- provide examples of relations with variables that vary directly, partially, or inversely
- solve problems dealing with direct, partial, or inverse variation

  A car travels at a constant speed and the distance it travels varies directly with time. It took Sam 6 minutes to travel 5 km. How long would it take Sam to travel 21 km?

*Continued*

---

<table>
<tr><td colspan="2" align="center">***Relations***</td></tr>
<tr><td align="center">**Acceptable Standard**</td><td align="center">**Standard of Excellence**</td></tr>
</table>

**Acceptable Standard**

- recognize graphs that illustrate direct, partial, or inverse variation

  For each of the following graphs, determine whether it is an example of direct, partial, or inverse variation.
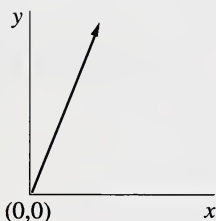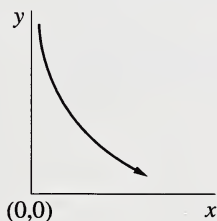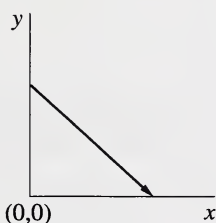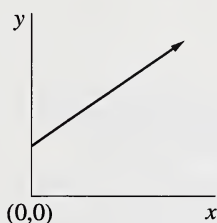
  

- recognize relations with variables that vary directly, partially, or inversely

  Determine whether each of the following examples is a direct, partial, or inverse variation:

  a. The cost of renting a hall is $250 plus $15 per person.

  b. Chris makes $5.50 an hour.

  c. The size of your piece of pie and the number of people sharing the pie.

**Standard of Excellence**

- solve problems dealing with direct, partial, or inverse variation, given a table of values or a graph

  The following table of values describes a relation between two variables.

  | $x$ | $y$ |
  |-----|-----|
  | 4   | 12  |
  | 6   | 18  |
  | 10  | 30  |

  a. What type of variation is this?
  b. Write a rule or give an equation that defines this variation.

| Geometry |
|---|

### General Outcome

*Students can describe the properties of the angles formed when parallel lines are intersected by a transversal.*

*Students can specify conditions under which triangles may be similar or congruent, and use these conditions to solve problems.*

| Acceptable Standard | Standard of Excellence |
|---|---|
| *The student can:* | *The student can also:* |
| • use the language associated with parallel lines (transversal, corresponding angles, alternate angles, interior angles) <br> • calculate the measure of vertically opposite angles | • provide a logical argument showing that vertically opposite angles are equal |
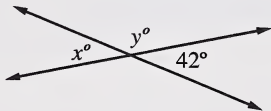
Find the missing measurements.



Find the missing measurements.



*Continued*

*Continued*

## Geometry

| **Acceptable Standard** | **Standard of Excellence** |
|---|---|

**Acceptable Standard**

- identify related pairs of angles formed by parallel lines intersected by a transversal



Name **all** pairs of corresponding, alternate, interior, and opposite angles.

- calculate the measure of any combination of corresponding, alternate, interior, and vertically opposite angles

Determine the value for the angles indicated by the letters in the following diagram:



*Continued*

*Continued*

| Geometry | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |
| • specify the conditions necessary for triangles to be congruent<br><br>Using a ruler and protractor, construct triangles given the following information.<br><br>Sides of 3 cm, 8 cm, and 10 cm. Determine if other triangles with these same dimensions have different measures for the angles.<br><br>Sides of 10 cm and 15 cm with an included angle of 60°. Determine if other triangles with these same dimensions and angle measure have different dimensions and angle measures for the other sides and angles.<br><br>Angles of 30° and 45° with an included side of 12 cm. Determine if other triangles with these same angle measures and side length have different dimensions and angle measure for the other sides and angle.<br><br>Angles of 30°, 60°, and 90°. Determine if other triangles with these same angle measures have different dimensions for the sides. | • summarize the conditions necessary to determine congruent triangles |
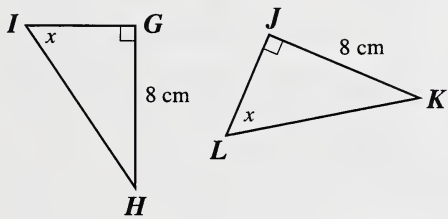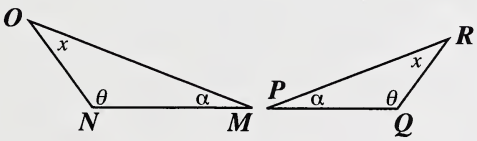
*Continued*

*Continued*

| Geometry |
|---|

| **Acceptable Standard** | **Standard of Excellence** |
|---|---|
| • use information obtained from visual representations to determine if two triangles are congruent or not | • identify a postulate by which a pair of triangles can be shown to be congruent |

Determine if each pair of triangles below is congruent.  Justify your answer.

**Pair 1:**



**Pair 2:**



• recognize that the measures of corresponding angles of similar triangles are equal

• recognize that the sides of similar triangles are proportional

• list the corresponding sides and angles of similar triangles

• show that two triangles are similar by using the ratio of their corresponding sides or the measure of their angles

*Continued*

*Continued*

## Geometry

| Acceptable Standard | Standard of Excellence |
|---|---|
| | • demonstrate that congruent triangles are similar triangles of the same size |
| • solve problems involving similar triangles that model everyday occurrences where a diagram is provided | • solve problems involving similar triangles that model everyday occurrences |
| Solve for a missing side, given a labelled diagram | A laser beam of light is directed from the top of a 40 m building to a mirror on the ground 15 m from the base of the building. The beam reflects off the mirror to the top of a second building that is 45 m from the mirror. Using similar triangles, determine the height of the second building. |
| Find the length of side $B_1C_1$. | |
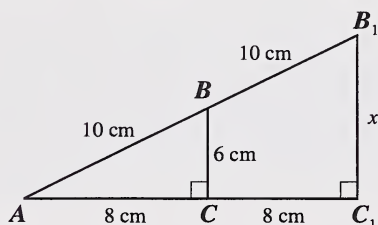


Determine the measure of a side of a pair of similar triangles that contains a pair of vertically opposite angles

Find the length of side $y$.

| Statistics |
|---|
| **General Outcome** |
| *Students can organize, analyze, and present data collected from a suitable sample of a population to draw inferences about this population.* |

| Acceptable Standard | Standard of Excellence |
|---|---|
| *The student can:* | *The student can also:* |
| • formulate a question that can be used in a survey in order to draw inferences about a real-world situation | |
| • formulate and write a hypothesis for the situation | |
| • identify the population and its characteristics | |
| • determine a sample that adequately and accurately represents the population | |
| • determine possible biases for a given sample, making reference to type of sample, sample size, and randomness | |
| • discuss the advantages and disadvantages of a chosen method of data collection | |
| • construct a frequency table, stem-and-leaf plot, box plot, and appropriate graphs (histogram, bar graph, pictograph, frequency polygon, line graph, and circle graph) given a set of data | |
| • calculate measures of central tendency | • choose appropriate measures of central tendency for a given set of data |

*Continued*

*Continued*

| Statistics | |
|---|---|
| **Acceptable Standard** | **Standard of Excellence** |
| • interpret stem-and-leaf plots, box plots, and graphs | • draw and defend conclusions based on the analysis of data organized in a stem-and-leaf plot, box plot, or graph |
| | Three gardeners are trying to determine who, on average, grows the heaviest tomatoes. They decide to take samples from their garden plots for comparison purposes. All three samples have the same mean mass, but different medians and ranges. Based on the box plots below, determine who grows the heaviest tomatoes. |
| | Gardener A ✱————[ [ ]————✱<br>Gardener B ✱————[] ————✱<br>Gardener C ✱————[ [ ]————✱<br>100 120 140 160 180 200 220 240 260 280 300<br>**Mass (grams)** |
| | Under which conditions would you prefer tomatoes produced by Gardener A, B, or C? |
| • make inferences about a population, based on the results from a survey<br>• communicate the design and results of a survey | • defend or oppose an inference that is made about a population, using the results of a survey |
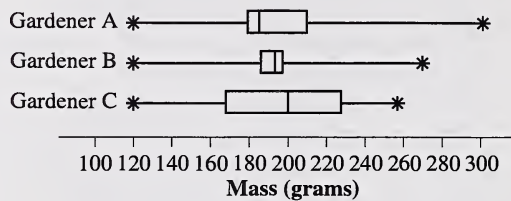
*Administering and Scoring the Assessment Components*

• *End-of-Course Exam*
• *Performance Assessment*
• *Project*
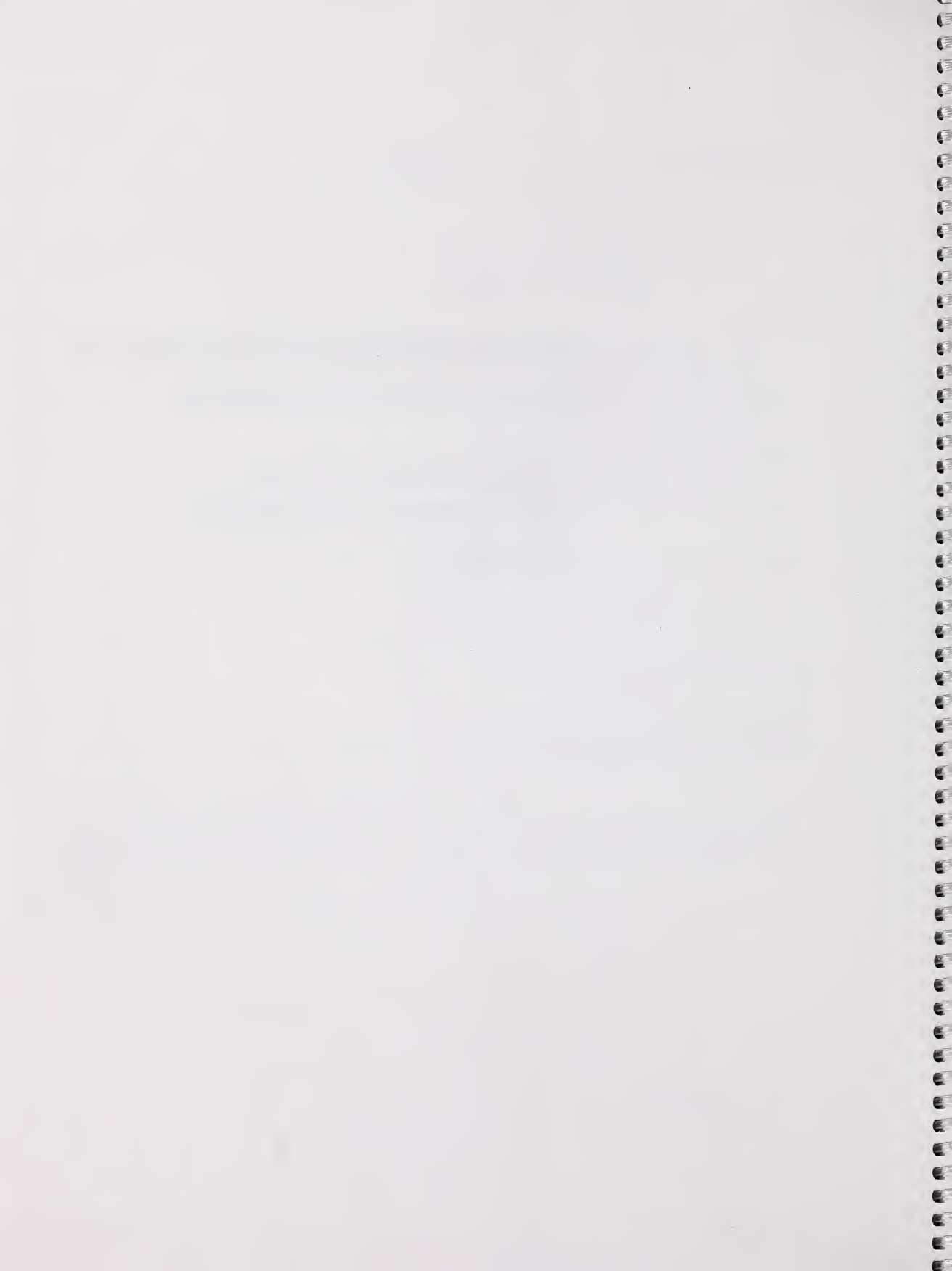
# End-of-Course Exam

**Administration**

This component takes 1.5 hours, with an additional 0.5 hours for students requiring extra time, and is a closed-book examination.

Students are supplied with a formula sheet but need to supply their own scientific calculator. They will also need a scorable answer sheet, which can be machine scored if the technology exists.

To assist students in maximizing performance and as a practice of fair assessment, it is suggested that the scoring criteria (on the next page) for the written-response component be shared with students before administration of the exam.

To assist you in marking and scoring students' answers, selected- and numerical-response scoring keys are provided below. To assist you in scoring the written-response questions, scoring criteria and sample solutions follow.

Examples of student work illustrating scoring criteria for the two written-response questions can be found in the *Mathematics 13 Examples of Students' Responses* document.

**Selected-Response Key**

| | | | |
|---|---|---|---|
| 1. C | 8. D | 15. B | 22. D |
| 2. A | 9. A | 16. A | 23. C |
| 3. B | 10. D | 17. D | 24. C |
| 4. A | 11. C | 18. C | 25. B |
| 5. B | 12. B | 19. B | 26. B |
| 6. A | 13. C | 20. C | 27. C |
| 7. D | 14. D | 21. A | 28. A |

**Numerical-Response Key**

1. 35.7
2. 13.1
3. 26
4. 2
5. 9
6. 18
7. 5

**Note: The *Student Booklet* contains instructions for answering numerical-response questions on answer sheets with prepared boxes. If this type of answer sheet is not available, these pages should be replaced with appropriate instructions for your students.**

*Mathematics 13*

# Scoring Criteria for Written Responses 1 and 2

| Mark | Scoring Criteria |
|---|---|
| **5** | *The student*<br>• provides a correct and complete solution<br>• demonstrates an understanding of the concepts and procedures related to the task<br>• provides communication that is clear, understandable, and logically organized |
| **4** | *The student*<br>• provides a complete solution with a few minor errors present<br>• demonstrates an understanding of the major concepts and procedures, although a small part may be missing or contain minor errors<br>• provides communication that is understandable and logically organized, but may contain a flaw or lack some clarity |
| **3** | *The student*<br>• provides a complete response that demonstrates a correct major step in the solution to the problem<br>• demonstrates some correct support or understanding of the major step<br>• communicates understandable reasons for solutions |
| **2** | *The student*<br>• provides a partial solution that is complete and correct<br>• demonstrates some support and/or understanding for the partial solution<br>• provides communication that is consistent with the task |
| **1** | *The student*<br>• makes a significant start that could have led to the successful conclusion of a major step<br>• provides a correct answer with no supporting work **or** provides a statement or procedure that shows some understanding of the process or steps involved<br>• provides communication that is inadequate to the task |
| **0** | *The student*<br>• presents work that is off topic<br>• presents a blank paper |

**Note:** Minor error means
- calculation error
- communication error
- missing or incorrect units, where applicable
- reasoning/logic error

# Solution for Written Response 1

*Use the following information to answer the next question.*

Bart is planning to refinish a wall, by either wallpapering or texture-painting it. The wall measures 4.9 m by 2.3 m and will be covered completely.

A roll of wallpaper costs $15.85 and covers an area of 2.5 m². Texture paint is sold in a 4 L container, which covers 4.2 m² and costs $23.50. Assume that wallpaper comes in **full** rolls only and that texture paint can be bought only in **full** 4 L containers.

**Written Response – 5 marks**

**1.** What is the most economical way for Bart to refinish the wall? Support your answer with calculations showing the actual cost of refinishing the wall both ways.

---

**A SOLUTION**

Surface area of wall: $4.9 \text{ m} \times 2.3 \text{ m} = 11.27 \text{ m}^2$

**Method I—Wallpaper**

Number of rolls:
$$\frac{11.27 \text{ m}^2}{2.5 \text{ m}^2 / \text{roll}} = 4.51 \text{ rolls}$$

∴ 5 rolls need to purchased

**Cost of wallpaper:**
5 rolls × $15.85/roll = $79.25

**Method II—Texture Paint**

Number of 4 L cans:
$$\frac{11.27 \text{ m}^2}{4.2 \text{ m}^2 / \text{L}} = 2.68 \text{ L}$$

∴ 3 cans of paint need to be purchased

**Cost of texture paint:**
3 rolls × $23.50/roll = $70.50

Texture-painting the wall will be more economical because wallpaper costs $79.25 and texture paint costs $70.50.

# Solution for Written Response 2

*Use the following information to answer the next question.*

Kelly's project requires her to change the design of a label, from a square whose side is $x$ centimetres, to a rectangle with a shortened width of $(x - 2)$ cm and area of $(x^2 + 4x - 12)$ cm$^2$, as shown below.

**Square**

$x$

$x$

**Rectangle**

$x - 2$

$x^2 + 4x - 12$

$l$

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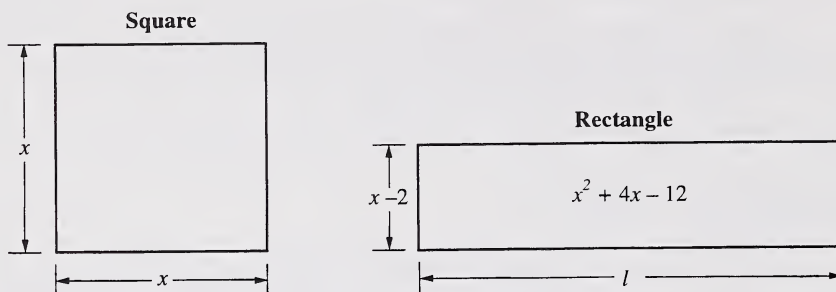
**Written Response – 5 marks**

**2.** **a.** Kelly takes the first steps to find the length, $l$, of the rectangle in terms of $x$ centimetres, by factoring the area of the rectangle. Complete her work and state the length.

$$\text{Area} = lw$$
$$A = x^2 + 4x - 12$$
$$A = (\quad\quad)(x - 2)$$

---

## A SOLUTION

If the area is equal to the length times the width, and the width is equal to $(x - 2)$ cm, then $(x + 6)$ is the length of the rectangle in centimetres.

Area $= (x + 6)(x - 2)$

The length of the rectangle is $(x + 6)$ cm.

---

*Continued*

*Continued*

**b.** Kelly compares the length of the rectangle to a side of the square. State the difference between the length of the rectangle and the length of a side of the square.

---

**A SOLUTION**

The length of the rectangle is 6 cm longer than a side of the square.

---

**c.** Kelly is told that the **areas** of the two labels have to be equal. She correctly determines that this will occur when $x = 3$. Explain why her reasoning is correct, and demonstrate that the areas would be different for other values of $x$.

---

**A SOLUTION**

**Part 1** Kelly is correct because when the value of $x = 3$ is selected, the dimensions of the square are 3 cm $\times$ 3 cm and the dimensions of the rectangle are 9 cm $\times$ 1 cm, both giving an area of 9 cm$^2$.

**OR** Kelly is correct, because the area of the square is $x^2$ and the area of the rectangle is $x^2 + 4x - 12$, and both areas are equal to 9 cm$^2$ when a value of $x = 3$ is substituted.

**Part 2** Values of $x$ that yield areas that are different are illustrated in the table below.

---

| | Square | Rectangle |
|---|---|---|
| $x$ (cm) | $x^2$ (cm$^2$) | $x^2 + 4x - 12$ (cm$^2$) |
| 4 | 16 | 20 |
| 5 | 25 | 33 |
| 6 | 36 | 48 |
| $N$ | $N^2$ | $N^2 + 4N - 12$ |

# *Performance Assessment*

## *Administration*

The performance assessment tasks are designed to assess aspects of mathematics that cannot be measured adequately by paper-and-pencil tests. These activities engage students in tasks that allow for a variety of strategies to be used, such as the use of manipulatives and the collection of information using other techniques. Each activity permits the assessment of integrated learning across and within subject areas, involves the active participation of the students, challenges students to solve problems, provides experience in communicating knowledge and skills, and provides a more complete picture of what students know and are capable of doing.

This component can be completed in a time of 30–40 minutes for each task at different sittings throughout the course, or in 90 minutes (with an additional 30 minutes if required) at one particular sitting. Tasks can be done in any order, and students must do the tasks on their own.

A list of prerequisite knowledge and skills, and the necessary materials to enable students to execute each task, are included in this manual.

To assist students in planning their solutions, the generalized scoring criteria for tasks 1, 2, and 3 should be shared with them before administering the performance assessment.

To assist you in marking and scoring the students' answers, solutions for each task and the scoring criteria have been provided. As well, examples of student work, with applied criteria and commentary, are provided for the three performance assessment tasks in the *Mathematics 13 Examples of Students' Responses* document.

# Scoring Criteria for Performance Assessment Tasks 1, 2, and 3

| Mark | Scoring Criteria |
|:---:|---|
| **6** | *The student*<br>• provides a correct and complete solution<br>• demonstrates an understanding of the concepts and procedures related to the task<br>• provides communication that is clear, understandable, and logically organized |
| **5** | *The student*<br>• provides a correct and complete solution, with a few minor errors<br>• demonstrates a clear understanding of all the major concepts and procedures<br>• provides communication that is clear, understandable, and logically organized, although minor flaws may exist |
| **4** | *The student*<br>• provides a complete solution, but some concepts or ideas are missing or misrepresented, and a few minor errors may be present<br>• demonstrates a correct understanding of the majority of major concepts and procedures<br>• provides communication that is understandable and logically organized, but may contain a flaw or lack some clarity |
| **3** | *The student*<br>• provides a complete response that demonstrates a correct major step in the solution to the problem<br>• demonstrates some correct support or understanding of the major step<br>• communicates understandable reasons for solutions |
| **2** | *The student*<br>• provides a partial solution that is complete and correct<br>• demonstrates some support and/or understanding for the partial solution<br>• provides communication that is consistent with the task |
| **1** | *The student*<br>• makes a significant start that could have led to the successful conclusion of a major step<br>• provides a correct answer with no supporting work **or** provides a statement or procedure that shows some understanding of the process or steps involved<br>• provides communication that is inadequate to the task |
| **0** | *The student*<br>• presents work that is off topic<br>• presents a blank paper |

**Note:** Minor error means
  • calculation error
  • communication error
  • missing or incorrect units, where applicable
  • reasoning/logic error

# Task 1:  Toothpick Triangle Patterns

## Student Task and Solution

***Performance Assessment General Outcome***

- this task requires students to discover a pattern or relation and connect variables, tables of values, and an equation to graphical representations

***Specific Prerequisite Knowledge and Skills***

- knowledge of graphing points, drawing lines
- knowledge of $y$-intercepts, skill in determining slope and writing equations in the form $y = mx + b$.
- knowledge of simple ratio calculations

***Performance***

- individual

***Time Required to Complete the Task***

- 30–40 minutes

***Materials Required***

- 15 toothpicks
- ruler

***Sample Solution***

A sample solution for the performance assessment is shown on the next page and can be used to familiarize yourself with a possible approach students might use.

*Continued*

*Continued*

An equilateral triangle can be formed using 3 toothpicks.
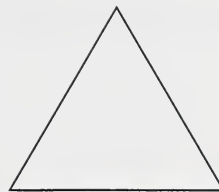
One triangle, which requires 3 toothpicks to build, is shown below.  Additional equilateral triangles can be formed by adding more toothpicks to the original triangle, as shown.
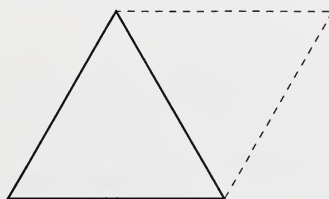
**Note:**    Do not form a pattern of triangles that represents a closed figure (with interim angles less than 180°), such as shown below.
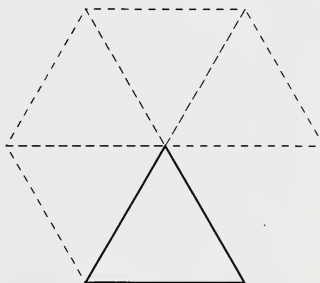
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*Continued*

**A.** Using your toothpicks, continue this construction to complete the chart below.
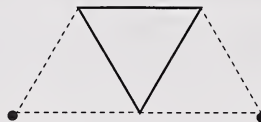


| Number of congruent equilateral triangles | 1 | 2 | 3 | 4· | 5 | 6 |
|---|---|---|---|---|---|---|
| Number of toothpicks required | *3* | *5* | *7* | *9* | *11* | *13* |

**B. i.** Represent the information contained in your table in graph form, using the grid below.
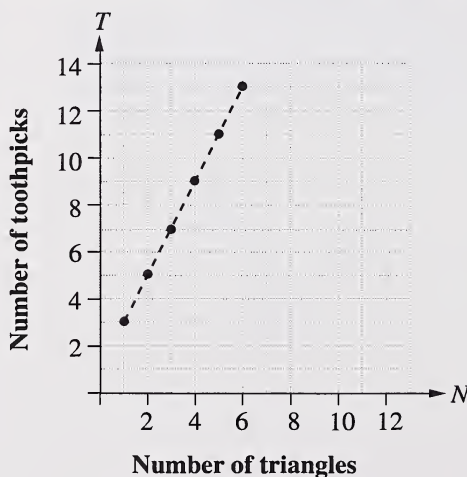


**ii.** Explain why a mathematician who was given this problem said that "the graph of this relation should not be a solid line, but, rather, specific points."

*There should only be points, since only whole toothpicks are used and whole triangles are formed.*

*Continued*

*Continued*

**C.** From either the table or the graph, determine the slope and *y*-intercept of this linear relation.

*The slope is 2.*
*The y-intercept is (0, 1).*

**D. i.** Find the equation that represents this linear relation and use it to determine the number of toothpicks required to make 10 triangles.

*The equation of the linear relation is y = 2x + 1.*
*To make 10 triangles requires 21 toothpicks.*

**ii.** Following this linear relation, how many triangles could you make from a box of 500 toothpicks? How many toothpicks would be left over?

*Using the linear relation, 249 triangles can be formed, with 1 toothpick left over.*

# Task 2: Volleyball Follies

## Student Task and Solution

**Performance Assessment General Outcome**

- this task requires students to draw an inference by organizing, presenting, and analyzing data supplied

**Specific Prerequisite Knowledge and Skills**

- knowledge of back-to-back stem-and-leaf plots
- knowledge of mean, median, mode, range (from stem-and-leaf plots)
- knowledge of 50% box plots

**Performance**

- individual

**Time Required to Complete the Task**

- 30–40 minutes

**Materials Required**

- ruler

**Sample Solution**

A sample solution for performance assessment is shown on the next page and can be used to familiarize yourself with a possible approach students might use.

As team manager for your school's volleyball team, you are scouting opposing teams and collecting statistics on their players. You decide to compare heights of the players of two other teams. The heights (in cm) of the players for these teams are given in the tables below.

| Springboks Player Heights (cm) | Antelopes Player Heights (cm) |
|---|---|
| 160 | 160 |
| 160 | 160 |
| 160 | 160 |
| 160 | 165 |
| 160 | 180 |
| 185 | 185 |
| 185 | 185 |
| 190 | 185 |
| 200 | 190 |
| 200 | 190 |

**A.** Organize this data in the back-to-back stem-and-leaf plot that has been started below.

| Springboks | | Antelopes |
|---|---|---|
| *0 0 0 0 0* | 16 | *0 0 0 5* |
| | 17 | |
| *5 5* | 18 | *0 5 5 5* |
| *0* | 19 | *0 0* |
| *0 0* | 20 | |

*Continued*

**B.** Calculate the mean, median, mode, and range of both sets of data.

|  | Springboks |  | Antelopes |
|---|---|---|---|
| mean | *176 cm* | mean | *176 cm* |
| median | *172.5 cm* | median | *182.5 cm* |
| mode | *160 cm* | mode | *160 cm, 185 cm* |
| range | *40 cm* | range | *30 cm* |

**C.** Organize the two sets of data into a 50% box-and-whisker plot below.

**Springboks**
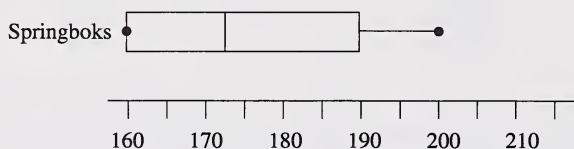


**Antelopes**
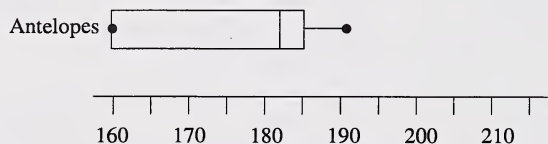


*Continued*

*Continued*

**D.** As manager, you are to report your scouting results to the coach. Based on your analysis, which of the two teams has the height advantage? Support your conclusion in paragraph form, using all calculated measures of central tendency and dispersion.

> *While the average player height is the same in both cases, the median height of the Antelopes is higher than the Springboks. While the Springboks may have the tallest player (200 cm), 50% of the Antelopes are 182.5 cm or taller compared with 50% of the Springboks who are 172.5 cm or taller. The Antelopes have more players of the same height that are taller (modes of 160 and 185 compared with the mode of 160 for the Springboks).*

> *Finally, the wider range of heights of the Springboks reflects the greater variation in their heights.*

> *Overall, the Antelopes have a height advantage as the team median and mode are greater than the median and mode of the Springboks.*

# Task 3: The Big Swallow

## Student Task and Solution

**Performance Assessment General Outcomes**

- this task requires students to determine equivalent forms and carry out operations in rational numbers
- this task requires students to use operations and equivalent forms of rational numbers to solve a direct variation problem

**Specific Prerequisite Knowledge and Skills**

- knowledge of arithmetic operations and applying operational skills

- knowledge of percent and problem-solving abilities using percent

- knowledge of and ability to apply direct variation to problem-solving setting

**Performance**

- individual

**Time Required to Complete the Task**

- 30–40 minutes

**Materials Required**

- calculator, pencil

**Sample Solution**

A sample solution for the performance assessment is shown on the next page and can be used to familiarize yourself with a possible approach students might use.

*Continued*

*Continued*

You are the manager of a local convenience store. One of the items that you sell is a flavoured ice drink called "**The Big Swallow**."



The Big Swallow
80 cents
Regular Price

Your **cost** for the drink and cups is always 30¢ per serving.
The difference between your **cost** and your **selling price** is your **profit**.

**A.** You normally sell the Big Swallow for 80¢.

    **i.** Calculate the profit you make per serving.

       *profit = 80¢ – 30¢ = 50¢ per serving*

    **ii.** Calculate your total profit if you sell 500 servings in one week.

       *500 × $0.50 = $250.00*

*Continued*

*Continued*

**B.** To increase your profits, you decide to experiment with different types of discounts.

> **The Big Swallow**
> **10% off**
> **This Week Only**

    **i.** During one week, you decide to try a 10% discount off the regular price.

        **a.** Calculate your new selling price and profit per serving.

> *selling price = 80¢ – 8¢ = 72¢ per serving*
> *profit = 72¢ – 30¢ = 42¢ per serving*

        **b.** If you sold 620 servings in this week, what would the total amount of your profit be?

> *620 × $0.42 = $260.40*

    **ii.** During another week, you try a coupon special, where each drink costs 10¢ less with a coupon.



        **a.** Calculate your new selling price and profit per serving.

> *selling price = 80¢ – 10¢ = 70¢ per serving*
> *profit = 70¢ – 30¢ = 40¢ per serving*

        **b.** If you sold 650 servings in this week, what would the total amount of your profit be? Show your calculations.

> *650 × $0.40 = $260.00  profit*

*Continued*

**iii.** For a third week, you try a reward system, where frequent customers get a fourth drink free if they purchase three at the regular price within the same week.



Under this plan, you sell 800 servings, which includes 200 free fourth drinks. How much profit do you make?

*600 servings @ 50¢ profit = $300.00*
*200 servings @ 30¢ loss = ($60.00)*

*Total profit = $240.00*

**C.** Which of the three plans was the best from the point of view of making a profit?

*At the current sales levels, the most profit was derived from the 10% discount approach.*

*Continued*

*Continued*

**D.** Which of the three plans was the best from the point of view of the purchasers

- if they bought a single serving in a week?

  > *Single serving*
  > *10% discount*      *10¢ off*      *Frequent Swallower*
  > *72¢*               *70¢*       *80¢*
  >
  > *The 10¢ off is best.*

- if they bought four servings in a week?

  > *Four servings*
  > *$2.88*              *$2.80*       *$2.40*
  >
  > *The Frequent Swallower special is best.*

**E.** If you wanted to continue with the Frequent Swallower plan, what is the minimum number of servings that you would have to sell in order to make $260 profit?

**Explain** and **use** a procedure to find this result.

> *A number of different approaches can be used to solve this, from ratio and proportion, to equations, to trial and error. At 868 (paid and free) servings, the vendor makes $260.40 profit.*
>
> **Note:** *Answers of 866 and 867 are also acceptable because the profit is $260.20 and $260.70, respectively (assuming that the pattern of 3 each then one free is maintained).*

# *Project*

## *Administration*

Projects are designed to assess aspects of mathematics that cannot be measured adequately by paper-and-pencil tests. This activity engages students in independent investigation, design creativity, the use of real-world data, and manipulatives. Students will also be engaged in communicating mathematical knowledge and will be given the opportunity to solve a problem using a variety of strategies and tools.

This component requires five periods of 60 minutes, or one week to complete, and should be carried out as a final project for the statistics unit.

Students will be expected to design, execute, interpret, and present their findings of a survey related to a topic of interest or question that generates numerical data. Each student should submit his or her findings in a well-organized written presentation. Groups of 2, 3, or 4 may be employed only during the design and data collection procedures of the project.

A list of guidelines and resources are included to provide the teacher and the students with a clear understanding of the content, thought processes, and communication that need to be considered in developing and completing the project.

To assist the teacher in marking and scoring the students' projects, the scoring criteria and a project scoring sheet have been included. As well, examples of student work, with applied criteria and commentary, are provided for the project in the ***Mathematics 13 Samples of Students' Responses*** document.

## *Problem*

As a statistical analyst, you are required to:
    **a.** design a statistical experiment that would generate **numerical data**
    **b.** collect **numerical data** and make visual presentations of your findings
    **c.** analyze your data
    **d.** generalize and evaluate your statistical findings

For this project, you may collect your data individually or in groups of 2, 3, or 4, and complete your own written presentation within 5 periods (one week).

## *Resources*

• Technologies to collect, calculate, and present data
• Data sources (publications, databases, Statistics Canada, surveys, etc.)
• Presentation instruments and materials (graph paper, poster board, photographs, tapes, etc.)

## *Project Guidelines*

The following guidelines and timelines provide suggestions and clarifications for the project.

### 1. Design
(Periods 1 and 2)  .

- Students should identify a problem or question of interest that generates **numerical data** and for which the answer is best determined through statistical analysis. Allow students to make their own creative choices.

  For students having difficulty choosing a topic, a few suggestions are:

  i. Design a statistical experiment to verify a scientific question or problem, such as "What is the average sentence length in a book by a favourite author, and how can statistics be used to describe the author's style?"

  ii. What is the average number of red candies in popular candy packages (or raisins in cereal packages, etc.)?

  iii. Does a professional hockey player make more than a professional baseball, basketball, or football player?

  iv. Is there a difference between wages in part-time jobs for high school females as compared with high school males?

- Students should give reasons as to why they selected their topic.
- Students should formulate a hypothesis or predict a general outcome of the investigation (e.g., "I believe that 30% of the M&M packages contain yellow candies").
- Students should identify the population to be surveyed.
- Students should determine an appropriate method for selecting a representative sample of the population (e.g., every fifth candy or person, random ID or phone numbers).
- Students should evaluate the sampling method to ensure that bias has been eliminated (adequate sample size, clearly worded survey question, appropriate timing, etc.).

  **Note:** Upon completion of the above tasks, students will receive your signed approval on their Project Approval Sheet. Approved project must respect participant privacy and community standards.

### 2. Organization and Visual Presentation
(Periods 2 and 3)

- Students should conduct their survey and neatly record their results on tally sheets (which should be kept).
- Students should organize and display data, using two (or more) of the formats studied in Math 13:
  – stem-and-leaf plots
  – 50% box-and-whisker plots
  – histograms or other appropriate displays
    (e.g., circle graphs, pictographs, line graphs, scatterplots)
- Students should present and defend reasons for the methods they selected.

*Continued*

*Continued*

### 3. Analysis
(Periods 3 and 4)

- Students should analyze their data and visual presentations. The analysis should include measures of central tendency and dispersion such as
  - mean, median, mode
  - upper and lower extremes
  - range
  - upper and lower quartiles
  - interquartile range
  - outliers (whiskers)

Example Data: 10, 20, 30, 40, 50, 60, 70



- Students should draw and defend conclusions based upon the analysis of their data, including an inference about the general population.
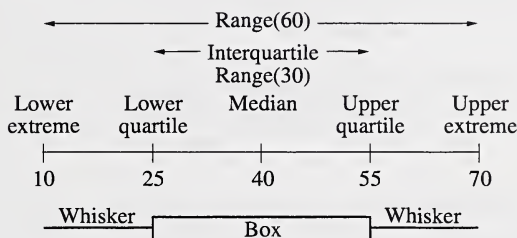
### 4. Generalization and Evaluation
(Periods 4 and 5)

- Students should defend or reject the original hypothesis or generalization with arguments based upon their data and analyses.
- Students should evaluate their project and its findings
  - error analysis, biases, difficulties, suitability
  - confidence of results
- Students should extend their findings to other populations and make further recommendations for future investigations that could improve the analysis of their topic.

## *Resources*

- Depending on the format in which students collect and present their findings, materials required for this project may vary.
- Technologies to collect, calculate, and present data may be used.
- Other sources (such as publications, databases, Statistics Canada) may be used to collect data.
- Teacher encouragement should be provided for students to be creative and imaginative in the topic identification, or collection and presentation of their data.

## *Project Scoring Criteria*

The scoring criteria rate the student's project across several categories. Sets of descriptions for judging and scoring student work are included under each category. The scores from each category can then be added to obtain a total mark for the entire project and recorded on the project scoring sheet. The scoring criteria that follows includes the categories of Design, Organization and Visual Presentation, Analysis, Generalization and Evaluation, and Communication. The criteria for each category are shared with students in the student booklet and should be applied holistically when scoring the project.

To assist in the marking process of each student's project, a project scoring sheet follows. The marks allotted in each of the five categories shall be added for a possible total of 30 marks and recorded on the class record form that follows. As well, examples of student work, with applied criteria and commentary, are provided for assessing the project and can be found in the *Mathematics 13 Examples of Students' Responses* document.

# Project Scoring Criteria

| Categories | Marks | Scoring Criteria |
|---|---|---|
| **Design**<br>• question, population, sample, hypothesis, method of collection | | *The student* |
| | 6 | • clearly identifies the problem and provides all steps and information necessary for conducting the project |
| | 5 | • clearly identifies the problem and provides all steps and information necessary for conducting the project, with a minor error |
| | 4 | • substantially identifies the problem and provides most steps and/or information necessary for conducting the project |
| | 3 | • reasonably identifies the problem, although some of the steps and information are flawed or missing |
| | 2 | • provides an unclear identification of the problem and the process; response may contain steps that are fragmented or missing |
| | 1 | • makes a significant start that shows some understanding of the process or information related to the problem |
| | 0 | • presents work that does not reflect any understanding of the statistical design process |

*Continued*

## Mathematics 13

*Continued*

| Categories | Marks | Scoring Criteria |
|---|---|---|
| **Organization and Visual Presentation**<br>• tally sheets, charts and/or graphs (at least two types) | | *The student* |
| | 6 | • chooses correct formats to organize and represent data. The charts, diagrams, or graphs are presented clearly, with supporting reasons for their choice |
| | 5 | • chooses correct formats to organize and represent data. All required charts, diagrams, or graphs are presented, but there may be minor errors in presentation |
| | 4 | • chooses correct formats to organize and represent data. Most charts, diagrams, or graphs are presented clearly and correctly |
| | 3 | • chooses correct formats to organize and represent data. Some charts, diagrams, or graphs contain major errors or are missing |
| | 2 | • presents a few charts, diagrams, or graphs, some of which are flawed |
| | 1 | • makes a significant start in presenting charts, diagrams, or graphs related to the problem |
| | 0 | • attempts charts, diagrams, or graphs that are not relevant |
| **Analysis**<br>• calculations/analysis of measures of central tendency and dispersion | | *The student* |
| | 6 | • analyzes and interprets correctly, thoroughly, and insightfully, using appropriate statistical procedures, measures, and calculations |
| | 5 | • analyzes the data effectively, using appropriate statistical procedures and calculations, but the analysis may contain few minor errors |
| | 4 | • analyzes and interprets major portions of the project and data reasonably, but small parts of the analysis are missing or flawed |
| | 3 | • makes a reasonable attempt to analyze the data, but the analysis contains major errors or lacks significant parts |
| | 2 | • makes a significant attempt to analyze some part of the data |
| | 1 | • makes a weak attempt to start an analysis or calculation relevant to the analysis |
| | 0 | • shows no relevant attempt to analyze the data |

*Continued*

*Continued*

| Categories | Marks | Scoring Criteria |
|---|---|---|
| **Generalization and Evaluation**<br>• conclusions, interpretations, and recommendations | 6<br><br>5<br><br>4<br><br>3<br><br>2<br><br>1<br><br>0 | *The student*<br>• provides generalizations and interpretations that are insightful, complete, and correct<br>• provides all required generalizations and interpretations, which are complete and correct<br>• provides some required generalizations and interpretations that are complete and correct<br>• provides some generalizations and interpretations that are weak but lead to a correct inference<br>• attempts to make generalizations and interpretations that are appropriate but flawed<br>• attempts to provide some generalization or interpretation<br>• attempts no generalization or interpretation |
| **Communication**<br>• appeal, organization, extension, consistency, and communication | 6<br><br>5<br><br><br>4<br><br><br>3<br><br>2<br><br><br>1<br><br>0 | *The student*<br>• provides detail, and communicates and presents the project effectively and clearly<br>• provides detail, communicates and presents the project effectively, but the communication contains a minor flaw<br>• communicates and presents the project well in general terms, but the communication lacks in specifics<br>• communicates and presents the project in an understandable but limited fashion<br>• provides visual and written communication that is lacking in organization and clarity or communication that is minimal<br>• attempts to communicate, and presents some part of the project<br>• provides communication that is missing or irrelevant |

*Mathematics 13*

# *Project Scoring Sheet*

Student Name: _____

Mathematics Course: _____

Instructor's Name: _____ Date: _____

Project Title: _____

## Suggested distribution of marks totalling 30

| Assessment criteria | Marks | Comments |
|---|---|---|
| Design | ____/6 | |
| Organization and Visual Presentation | ____/6 | |
| Analysis | ____/6 | |
| Generalization and Evaluation | ____/6 | |
| Communication | ____/6 | |

Total marks: _____/30 = _____ %

Other comments:

Student's comments:

Teacher's signature _____

Student's signature _____

# *Calculating and Recording Student Achievement*

- *Standards for Overall Performance on the Assessment*
- *Class Record Form*
- *Item Summaries*
- *Directing Words*
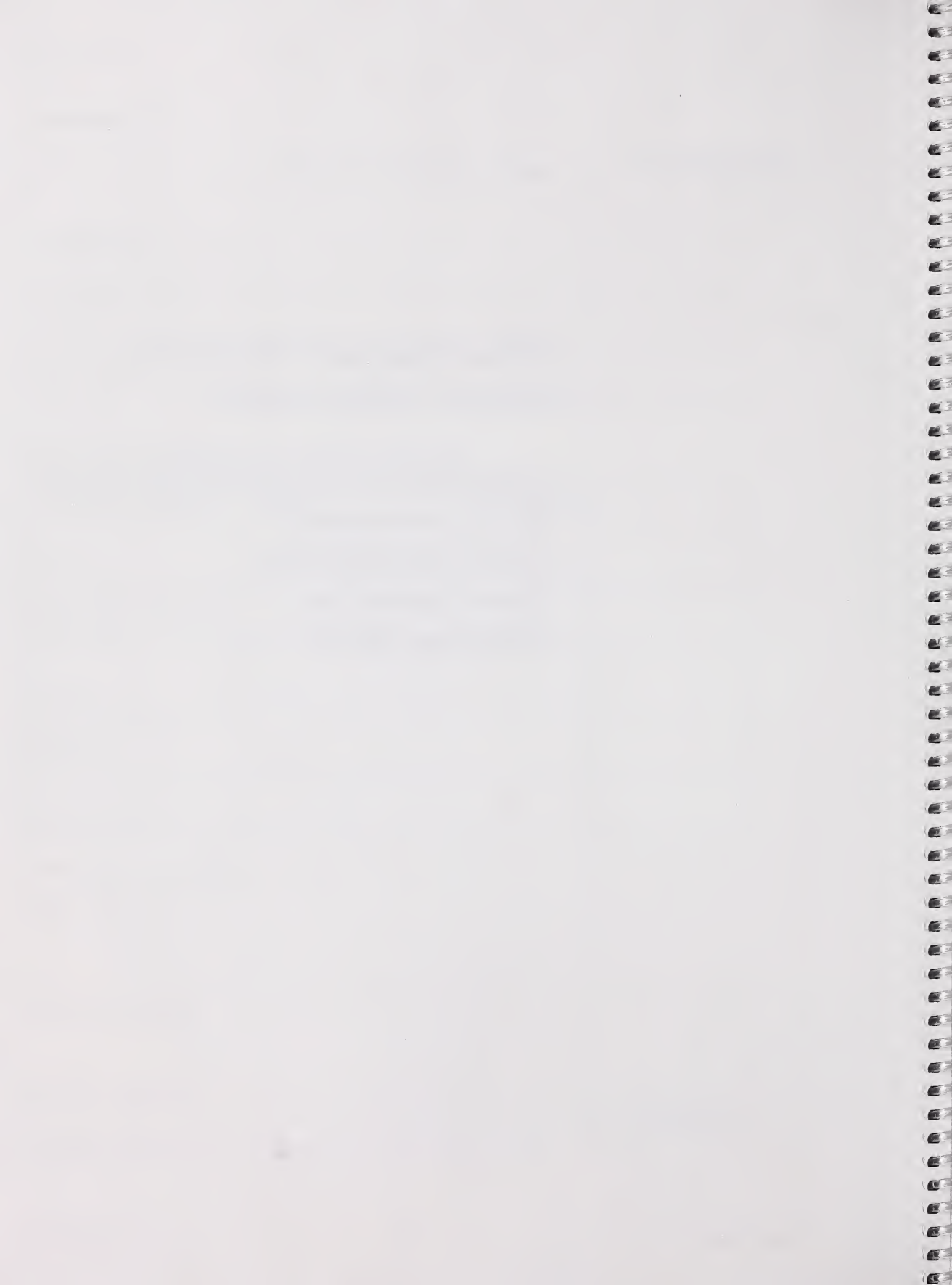
# Standards for Overall Performance on the Assessment

Scoring criteria for each component of the assessment are provided in the teacher instructions for each component. Assessment standards for the overall performance of a student on the whole package are as follows:

| Not Yet at Acceptable Standard (NS) | Acceptable Standard (AC) | Standard of Excellence (EX) |
|---|---|---|
| 0%–49% on the total score | 50%–79% on the total score | 80%–100% on the total score |

Standards for the assessment were established and validated using the scoring criteria outlined for each task. If the tasks are scored using different criteria or used for purposes other than assessing achievement at the end of Mathematics 13, these standards may not be appropriate.

The standards for performance on the assessment apply to all students. Professional judgement should be used to make adjustments to administration procedures so that special needs students are able to demonstrate their best work.

# Class Record Form

| Student Name | A End-of-Course Exam % $\dfrac{}{45} \times 100$ | B Performance Assessment $\dfrac{}{18} \times 100$ | C Project % $\dfrac{}{30} \times 100$ | 0.6A + 0.15B + 0.25C Overall weighted achievement % | Standard* |
|---|---|---|---|---|---|
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*The standards for this assessment package are as follows:

| Not Yet at Acceptable Standard (NS) | Acceptable Standard (AC) | Standard of Excellence (EX) |
|---|---|---|
| 0%–49% on the total score | 50%–79% on the total score | 80%–100% on the total score |

# *Item Summaries*

## *End-of-Course Exam*

| Item # | Unit | Curriculum Objective* | Taxonomy |
|---|---|---|---|
| SR 1 | NS | 1.1.1 | P |
| SR 2 | NS | 1.1/1.2 | P/PS |
| SR 3 | NS | 1.1.2 | P/PS |
| NR 1 | STAT | 2.3/2.5 | P/PS |
| SR 4 | REL | 2.1.2 | C/P |
| SR 5 | REL | 1.3/2.1 | P |
| NR 2 | REL | 2.1.1 | P |
| SR 6 | COG | 2.1 | P |
| SR 7 | COG | 1.4.1 | C/P |
| NR 3 | POLY | 1.2 | P/PS |
| SR 8 | REL | 1.3 | P |
| SR 9 | POLY | 2 | P |
| SR 10 | POLY | 3.2 | P |
| NR 4 | POLY | 1.1/3.1 | C/P |
| SR 11 | POLY | 3.5 | P |
| SR 12 | POLY | 3.3 | P |
| SR 13 | POLY | 4.1 | C |
| SR 14 | POLY | 4.4 | P |
| SR 15 | GEOM | 3 | C |
| SR 16 | GEOM | 2.1.1 | C/P |
| SR 17 | GEOM | 1.3 | C/P |
| SR 18 | GEOM | 1.2 | C/P |
| SR 19 | GEOM | 3.1 | C/P |
| NR5 | GEOM | 3.1 | P |
| NR6 | NS | 1.3.1 | PS |
| SR 20 | COG | 1.3 | P |
| SR 21 | COG | 1.1 | P |
| NR 7 | COG | 2.2 | C |
| SR 22 | COG | 1.1.3 | C |
| SR 23 | NS | 2.1 | P |
| SR 24 | STAT | 2.3/2.5 | C |
| SR 25 | STAT | 2.3/2.5 | C |
| SR 26 | STAT | 2.3 | C/P |
| SR 27 | STAT | 2.2 | C |
| SR 28 | STAT | 1.2 | C |
| | | | |
| WR 1 | NS | 1 | C/P/PS |
| WR 2 | POLY | 1.2/4.5.1 | C/P/PS |

| Question type | Marks | Percent Emphasis |
|---|---|---|
| Selected Response | 28 | 78 |
| Numerical Response | 7 | |
| Written Response | 10 | 22 |

| Taxonomy | Marks | Percent Emphasis |
|---|---|---|
| Conceptual Understanding | 14 | 31 |
| Procedural Understanding | 22 | 49 |
| Problem-Solving Understanding | 9 | 20 |

| | |
|---|---|
| C – Conceptual Understanding | |
| P – Procedural Understanding | |
| PS – Problem-Solving Understanding | |

| | |
|---|---|
| COG | – Coordinate Geometry |
| REL | – Relations |
| NS | – Number Skills |
| POLY | – Polynomials and Factoring |
| STAT | – Statistics |
| GEOM | – Geometry |

| | |
|---|---|
| SR | – Selected Response |
| NR | – Numerical Response |
| WR | – Written Response |

*These are curriculum objectives referred to by number in the Alberta Education Course of Studies for Mathematics 13.

*Continued*

*Mathematics 13*

*Continued*

## Performance Assessment

| Task | Value | Unit | Curriculum Objective* | Taxonomy |
|------|-------|------|----------------------|----------|
| 1 | 6 | COG | 1.1/1.3/2.1/2.2/2.4 | C/P/PS |
|   |   | REL | 1.1 |  |
| 2 | 6 | STAT | 2.2/2.3/2.4/2.5/2.6 | C/P/PS |
| 3 | 6 | NS | 1.1/1.2 | C/P/PS |

## Project

| Project | Value | Unit | Curriculum Objective* | Taxonomy |
|---------|-------|------|----------------------|----------|
| 1 | 30 | STAT | 1.1 to 2.6 | C/P/PS |

*These are curriculum objectives referred to by number in the Alberta Education Course of Studies for Mathematics 13.

# *Directing Words*

**Discuss**
The word "discuss" will not be used as a directing word on math and science assessments because it is not used consistently to mean a single activity.

*The following words are specific in meaning:*

**Contrast/Distinguish**
Point out the *differences* between two things that have similar or comparable natures.

**Compare**
Show the character or relative values of two things by pointing out their *similarities* and *differences*.

**Conclude**
State a logical end based on reasoning and/or evidence.

**Criticize**
Point out the *merits* and *demerits* of an item or issue.

**Define**
Provide the essential qualities or meaning of a word or concept. Make distinct and clear by marking out the limits.

**Describe**
Give a written account or represent the characteristics of something by a figure, model, or picture.

**Design/Plan**
Construct a plan; i.e., a detailed sequence of actions, for a specific purpose.

**Enumerate**
Specify one by one or list in concise form and according to some order.

**Evaluate**
Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages.

**Explain**
Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail.

**How**
Show in what manner or way, with what meaning.

**Hypothesize**
Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically.

### Identify
Recognize and select as having the characteristics of something.

### Illustrate
Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram.

### Infer
Form a generalization from sample data; arrive at a conclusion by reasoning from evidence.

### Interpret
Tell the meaning of something; present information in a new form that adds meaning to the original data.

### Justify/Show How
Show reasons for or give facts that support a position.

### Outline
Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps.

### Predict
Tell in advance on the basis of empirical evidence and/or logic.

### Prove
Establish the truth, validity, or genuineness of something by giving factual evidence or logical reasons.

### Relate
Show logical or causal connection between things.

### Solve
Give a solution for a problem; i.e., explanation in words and/or numbers.

### Summarize
Give a brief account of the main points.

### Trace
Give a step-by-step description of the development.

### Why
Show the cause, reason, or purpose.